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Dynamic Pricing in the Vehicle Ferry Industry

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Talk Overview

- Problem description
- General framework for integrating packing and pricing
- Packing models (Exact and Simulation based state definitions)
- Price acceptance model
- Results
- Conclusions and future work

PROBLEM DESCRIPTION

Problem description

Objective: derive a dynamic pricing policy that maximises the expected revenue from the sale of vehicle tickets on a ferry

- **Constraint:** Limited **capacity** which **depends on packing efficiency**
- Customers
  - Arrive at random during the **selling season** (beginning 6 months before departure)
  - Customer **willingness-to-pay** is dependent on time until departure and varies between vehicle types
  - Vehicles vary in **size**

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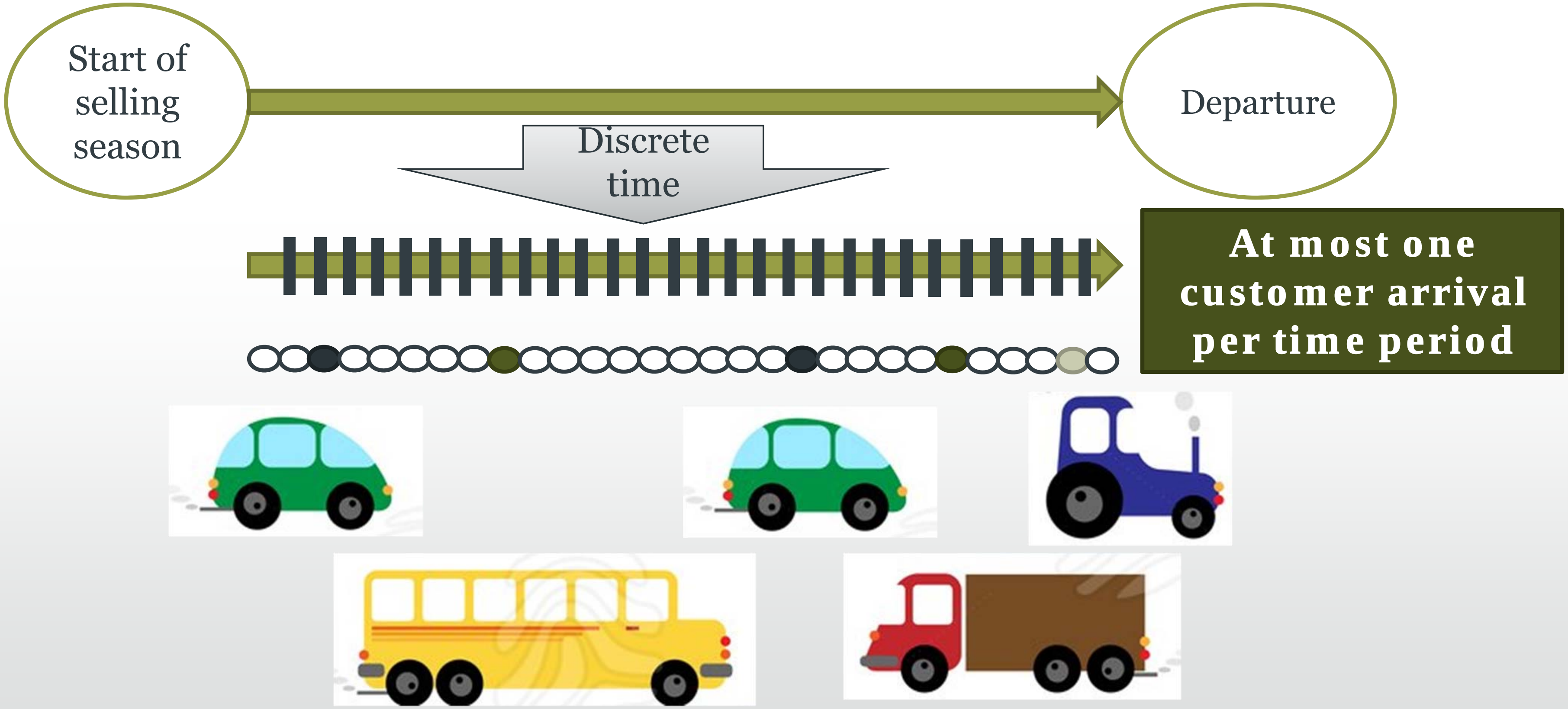
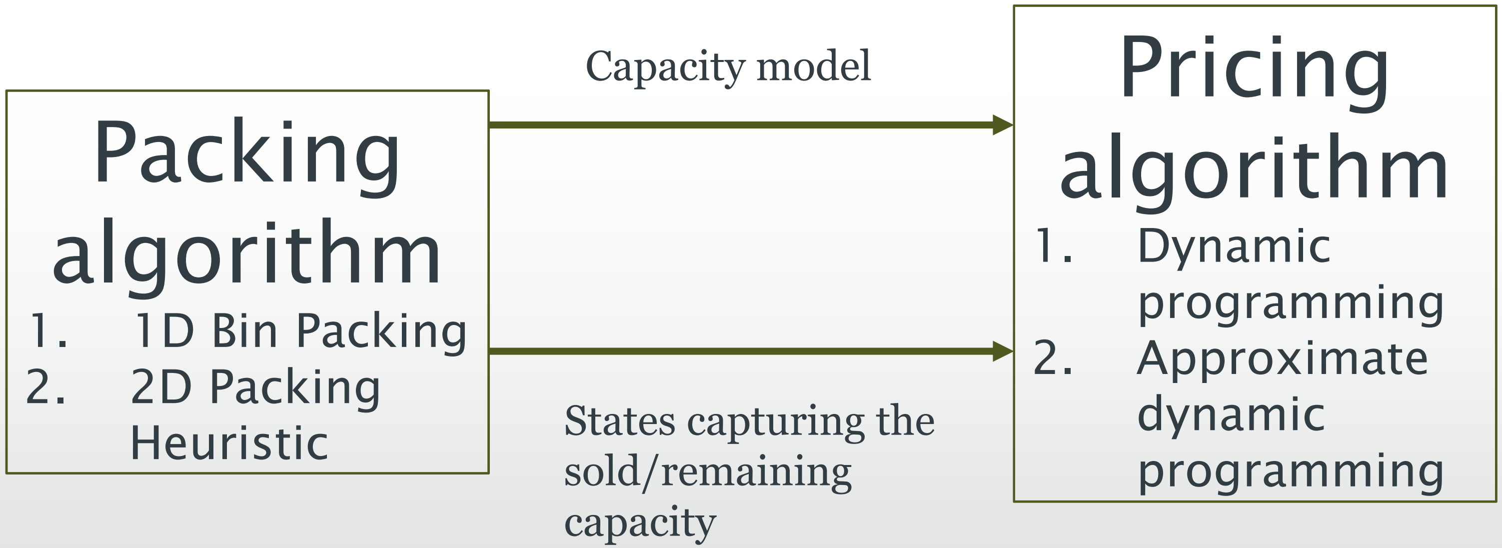
Selling Tickets



General framework for integrating packing and pricing



General framework for integrating packing and pricing

- Input variables
  - The state $\boldsymbol{s}$ at any given time interval captures the ferries remaining capacity for vehicles. The key question is how to define $\boldsymbol{s}$, we consider exact and approximate approaches
  - $\boldsymbol{s'}$ denotes the new remaining capacity state after one sale whilst in state $\boldsymbol{s}$
  - $\boldsymbol{V_t(s)}$ denotes the 'revenue-to-go' or the expected future revenue if the state is $\boldsymbol{s}$ at time $\boldsymbol{t}$
  - $\boldsymbol{\lambda_{t,i}}$ denotes the probability that a customer with vehicle type $\boldsymbol{i}$ arrives at time $\boldsymbol{t}$

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General framework for integrating packing and pricing

- Input functions
  - **Price acceptance function:** $\boldsymbol{\alpha}(\boldsymbol{i}, \boldsymbol{p}, \boldsymbol{t})$ returns the probability that a customer with vehicle type $\boldsymbol{i}$ will pay a price $\boldsymbol{p}$ at time $\boldsymbol{t}$
  - **Transition function:** $\boldsymbol{f}(\boldsymbol{s}, \boldsymbol{i})$ returns the remaining capacity capturing state $\boldsymbol{s}'$ if a customer with vehicle type $\boldsymbol{i}$ purchases a ticket at a time when the state is $\boldsymbol{s}$ (derived from packing models)

General framework for integrating packing and pricing

- Dynamic pricing formulation
  - The optimal dynamic pricing look-up-table policy can be derived by computing the Bellman equations by backwards recursion
  - In each state at each time 3 events can occur

    1. No customers arrive
    2. A customer arrives but does not purchase a ticket
    3. A customer arrives and purchases a ticket

$$V_t(s) = \max_{p \in P} \left\{ \left\{ \sum_{i \in I} \overset{(3)}{\lambda_{t,i} \{ \alpha(i,p,t)(p + V_{t-1}(f(s,i)))} + \overset{(2)}{\big(1 - \alpha(i,p,t)\big) V_{t-1}(s) \}} \right\} + \overset{(1)}{\lambda_{t,0} V_{t-1}(s)} \right\}$$

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Exact and Simulation-based state definitions

PACKING MODELS

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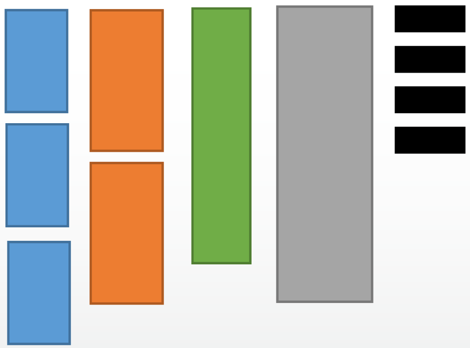
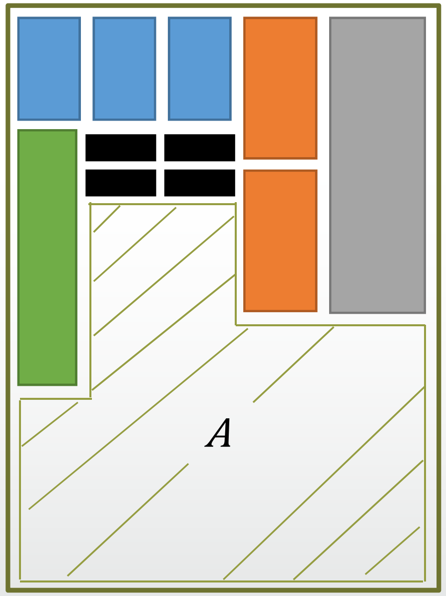
Exact and Simulation based state definitions

Exact

- 1-d bin packing model (optimal lane parking)
- State=count of vehicles of each type
- 1-dimension per vehicle type
- **Tickets sold state definition**



Simulation based



- 2-d packing heuristic (ignores lanes)
- State=remaining deck area per deck region
- 1-dimension per deck region
- **Remaining space state definition**

Transition functions for one vehicle type o sale

$state = 3,2,1,1,4$

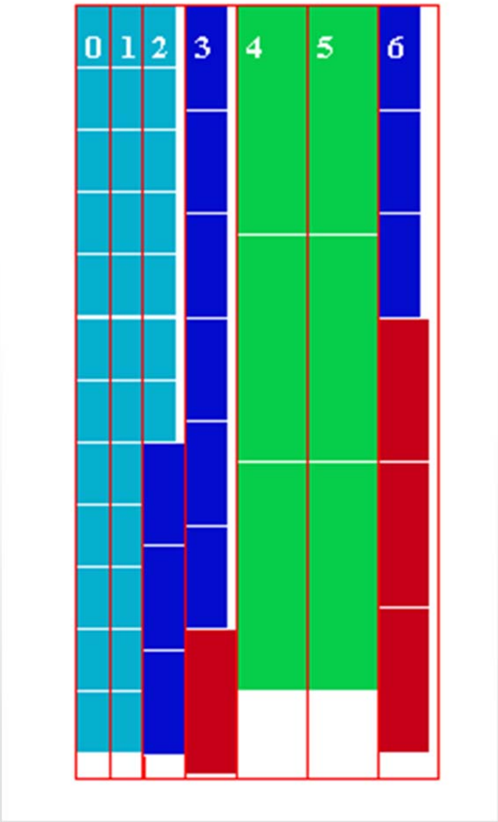
$f(\{3,2,1,1,4\},0) = \{3,2,1,1,4\} + \{1,0,0,0,0\} = \{4,2,1,1,4\}$

$state = A = 950.8, \qquad transition\ value = 26.2$

$f(950.8,0) = 950.8 - 26.2 = 924.6$

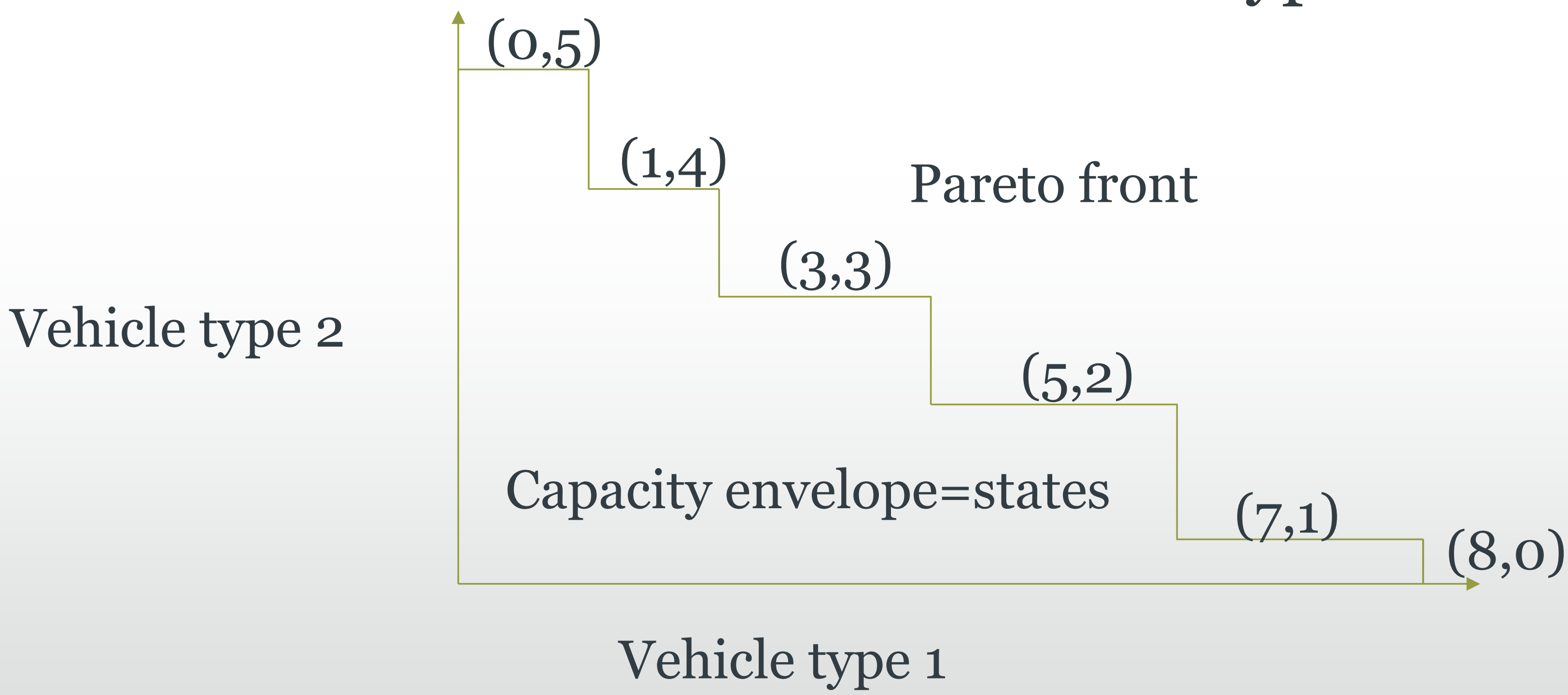
Exact (1D Bin Packing)

- Assumes that vehicles can be allocated to lanes that they fit within
- A fast IP formulation is used to enumerate all possible vehicles mixes



Pareto front of vehicle mixes

Vehicle type 2 > vehicle type 1



Loading Simulator

- Simulates the **vehicle ferry loading process** for a known set vehicles
- **2D packing problem** on the main deck, as not all vehicle types fit within the lanes
- **Sequential** weighted sum **loading algorithm** for placing vehicles (simulated annealing is used to tune the weights to increase packing efficiency)
- **Real world constraints**:
  - Manoeuvrability; lift access; mezzanine decks; drop trailers which are towed onto the ferry, parking gaps and also reverse gaps
- Purpose:
  - **Map** vehicle mix **states** to lower dimensional **remaining space states**
  - Generate efficient **2D packing solutions**
  - **Prevent overselling** (assuming 100% show rate)

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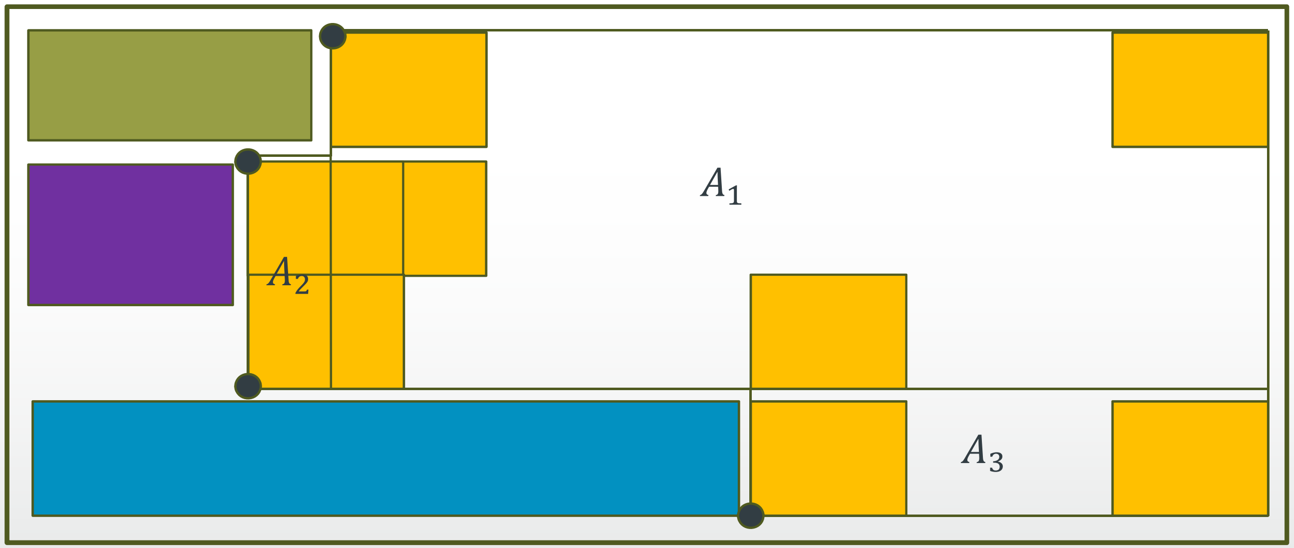
Mapping a vehicle mix state to a remaining space state



Available parking positions

Remaining area which is used to map vehicle mix state to remaining space state

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Simulation based approach state transition functions

- **Transition functions** specify the average amount of **area used by each vehicle** type including area lost due to staggered parking (parking loss)
- Transition functions are **derived from** the transitions that occur in a large sample of simulated vehicle loads
- The **loading efficiency** of each simulated vehicle load maximised via a simulated annealing algorithm

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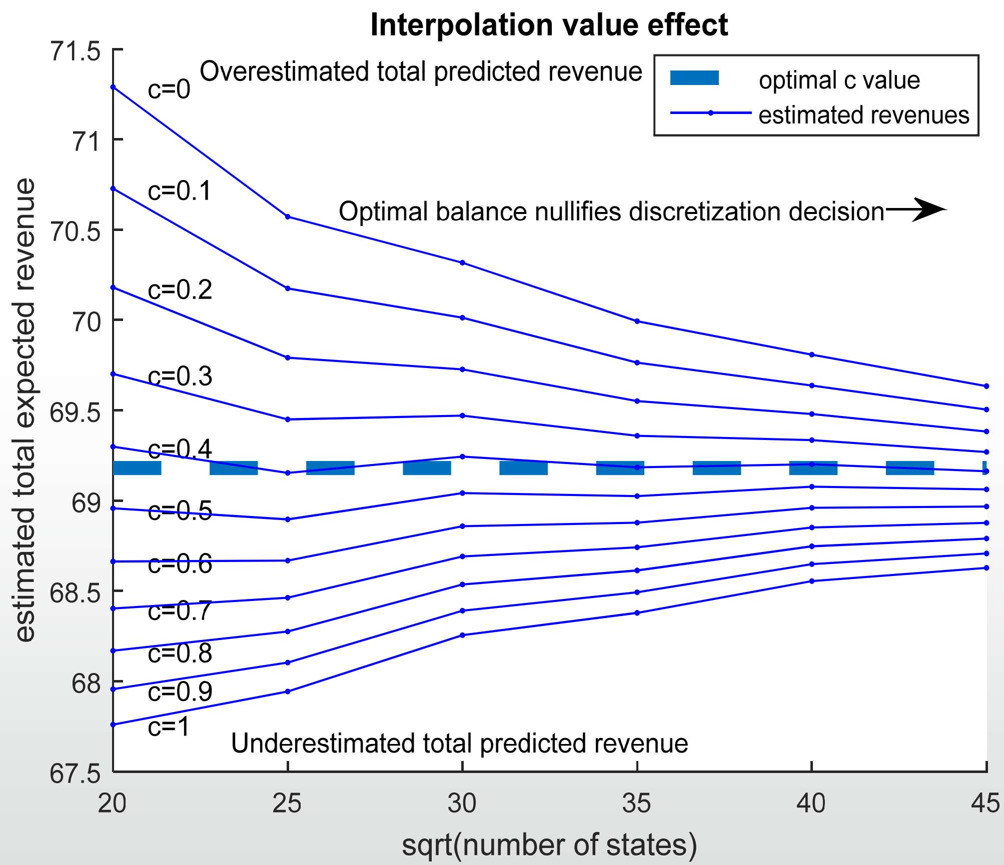
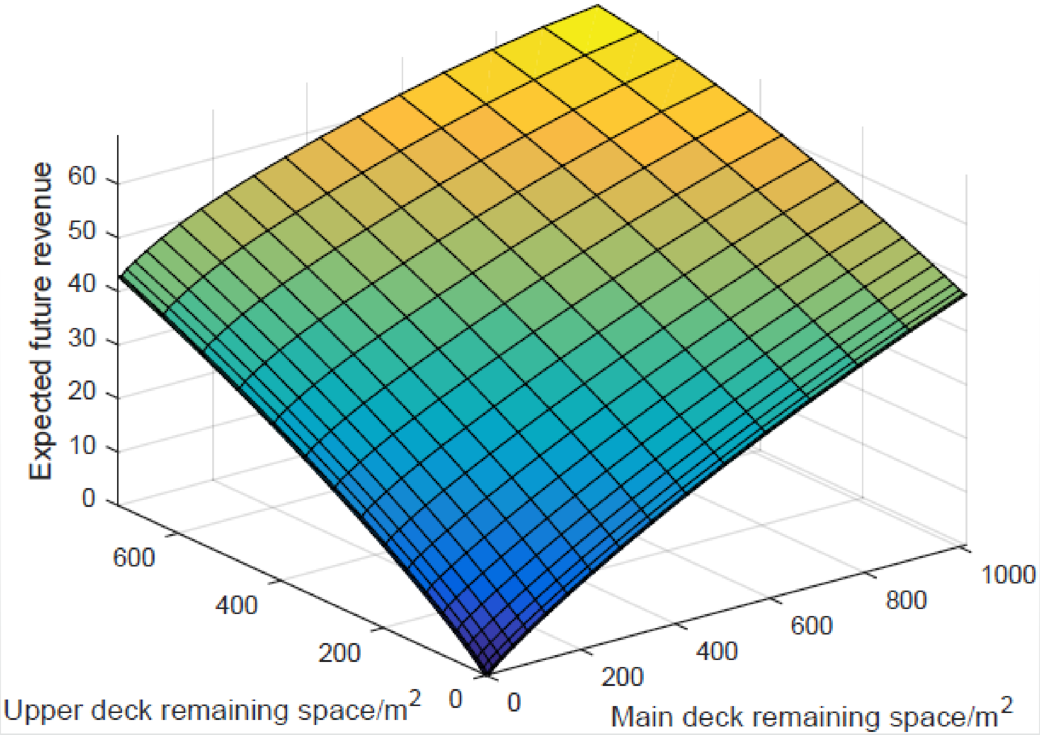
Approximating the value function (Simulation based approach)

- The remaining space is continuous but we solve the value function for a **discrete** set of **remaining space states**
- Transitions from the discrete states lead to **intermediate states**
- The values of intermediate states are **interpolated** from the values of neighbouring states



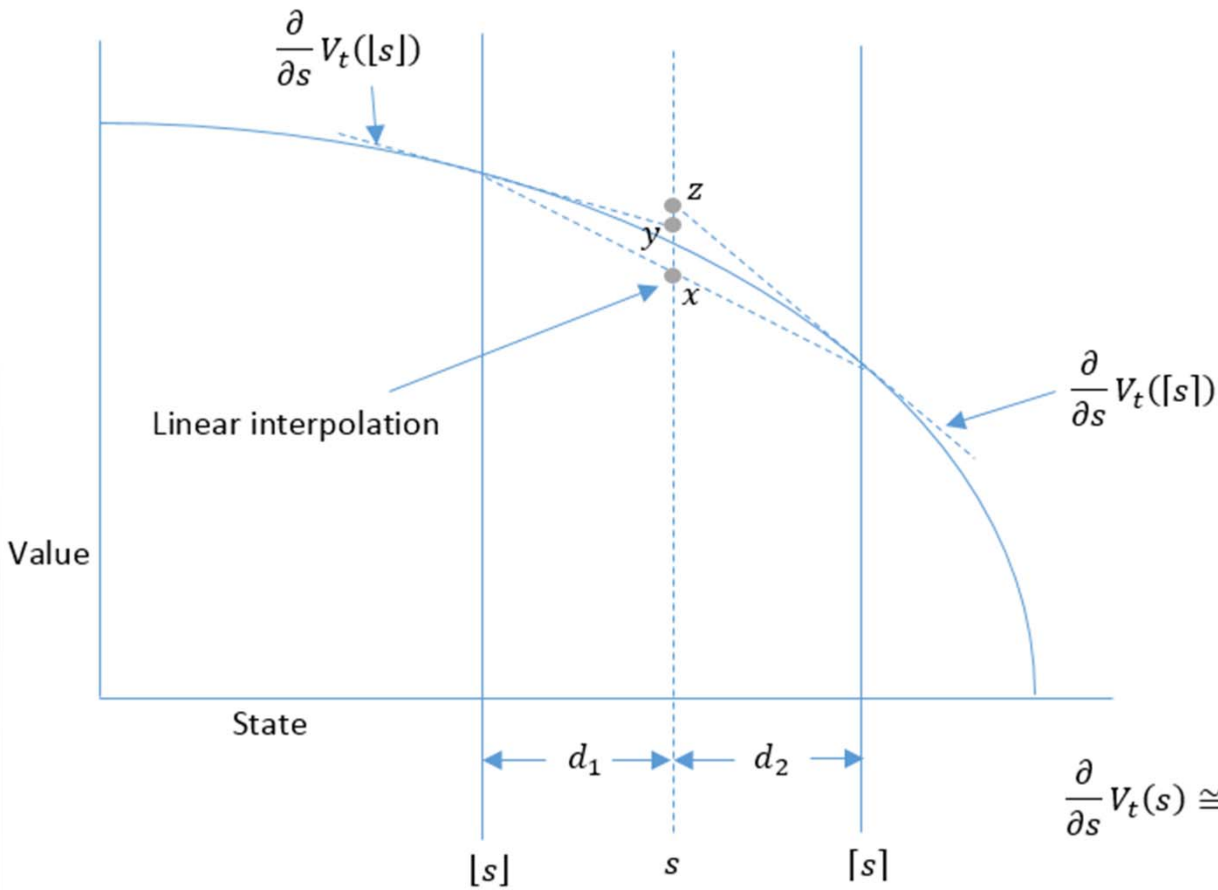
$$V_t(s) = c(linear\ interpolation) + (1 - c)(gradient\ based\ interpolation)$$

Concave structure of value function



The concave structure of the value function is exploited to speed up the solution time of the dynamic program

Value interpolation



$$x = d_2 V_t(\lfloor s \rfloor) + d_1 V_t(\lceil s \rceil)$$

$$y = V_t(\lfloor s \rfloor) + d_1 \frac{\partial}{\partial s} V_t(\lfloor s \rfloor)$$

$$z = V_t(\lceil s \rceil) - d_2 \frac{\partial}{\partial s} V_t(\lceil s \rceil)$$

$$V_t(s) = cx + (1 - c)(d_2 y + d_1 z)$$

$$\frac{\partial}{\partial s} V_t(s) \cong \frac{\big(V_t(s) - V_t(s-1)\big) + \big(V_t(s+1) - V_t(s)\big)}{2}$$

Price acceptance model (for both models)

- Accounts for bell shaped WTP distribution and monotonic time effects

(Price component) (Time component)

- $$\alpha(p,t) = cf\left(1-\left(\frac{1}{1+e^{-k\left(\frac{p}{pMax}-m\right)}}\right)\right)\times\left(a+(b-a)\left(1-\frac{t}{T}\right)^{c}\right)$$

- $$cf = \left(\frac{1}{\left(1-\left(\frac{1}{1+e^{k\cdot m}}\right)\right)}\right)$$

Parameter	Interpretation
a	The probability of price acceptance at the beginning of the selling season at price 0
b	The probability of price acceptance at the end of the selling season at price 0
c	Curvature of the effect of time on the probability of price acceptance
k	Steepness of the midpoint of the sigmoidal price part of the function
m	Relative position of the midpoint of the sigmoidal part of the function
pMax	Maximum price a random customer will pay

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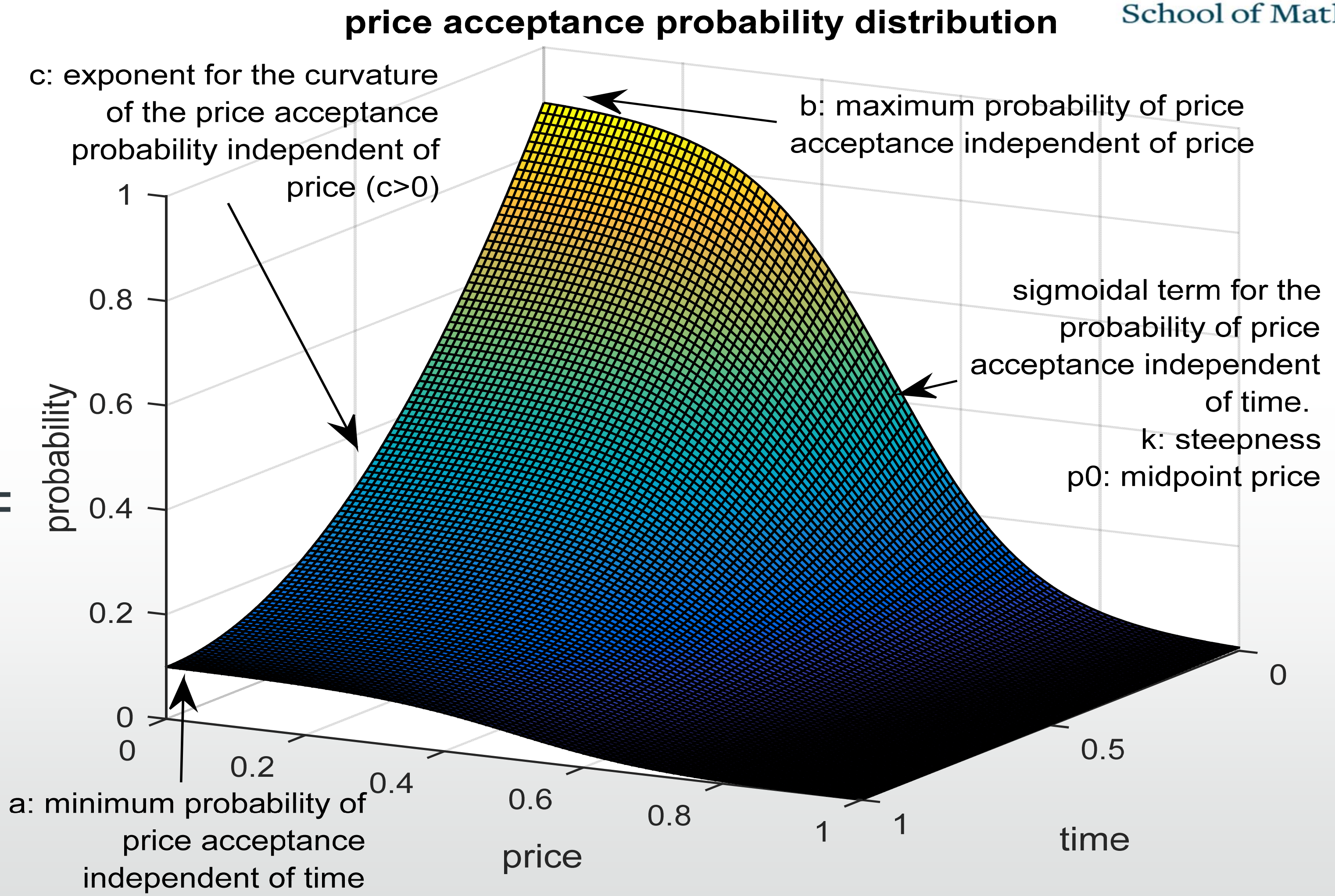
$$\alpha(i,p,t) =$$



Experiments

- Exact
  - The impact of
- Simulation based
  - 13 vehicle types, real ferry design with a car deck and a main deck with up to 2 mezzanine decks, $pMax(i) \propto area_i$, 3 deck configurations, 3 demand scenarios $(\lambda_{t,i})$
- Exact versus Simulation based
  - Simulation based approach implemented with 1D and 2D packing approaches in the Exact test instances

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Experiments

- Exact
  - 5 vehicle types, 3 lane types, $pMax(i) \propto \sqrt{length_i}$, T=1000
- Simulation based
  - 13 vehicle types, real ferry design with a car deck and a main deck with up to 2 mezzanine decks, $pMax(i) \propto area_i$, 3 deck configurations, 3 demand scenarios $(\lambda_{t,i})$
- Exact versus Simulation based
  - Simulation based approach implemented with 1D and 2D packing approaches in the Exact test instances

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Exact Experiment Parameters

The customers

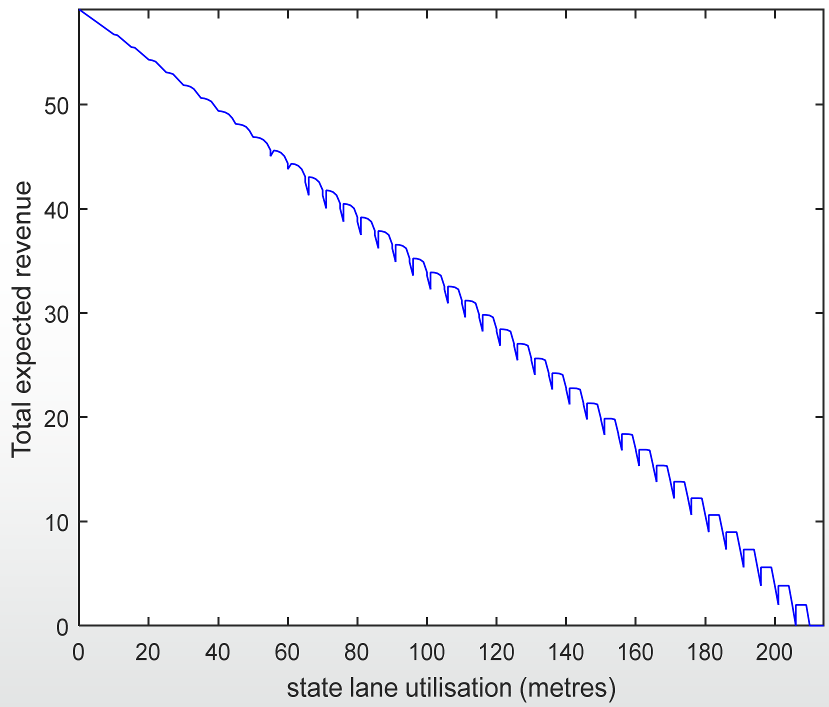
Vehicle type	Arrival rate	Max price (SQRT(L))	Length	Width	Height
1	0.4	1.732051	3	1.6	1.5
2	0.2	2.236068	5	2.3	1.5
3	0.15	2.645751	7	2.4	2.5
4	0.1	3	9	2.9	3
5	0.05	3.316625	11	3.4	4

The ferry

Lane type	quantity	Length	Width	Height
1	2	37.04	2.34	5
2	2	37.04	2.93	5
3	2	37.04	3.42	5

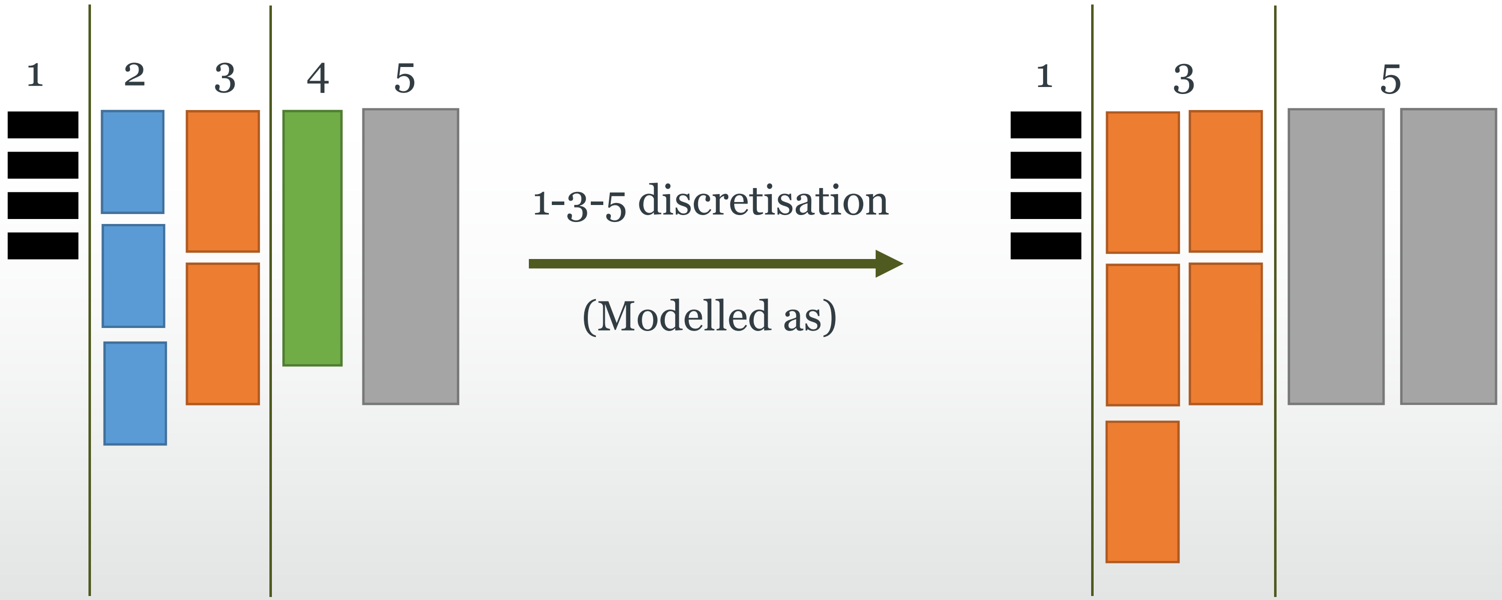
The interaction between packing and pricing

- X-axis: vehicle mix state sorted by total length of vehicles
- Y-axis: Total expected revenue
- Interpretation: future profit does not strictly monotonically increase with remaining lane space
- This is due to packing effects:
  - Some vehicle mixes lead to unusable gaps at the ends of lanes
  - Our framework will increase the price of sales that lead to such bad states



Vehicle type discretisation

To **reduce the dimensionality** of the problem when using the Exact approach vehicles can be mapped to fewer categories



$state = 4{,}3{,}2{,}1{,}1$

$state = 4{,}5{,}2$

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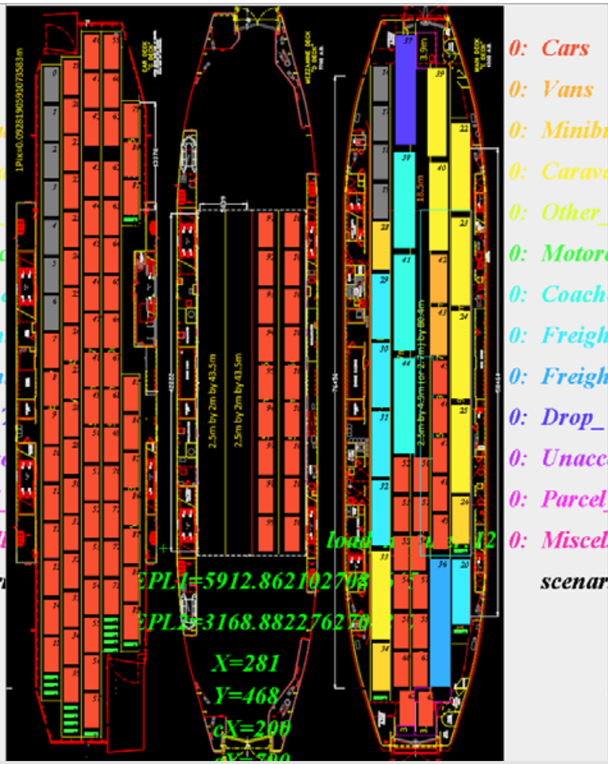
Vehicle type discretisation results

Vehicle type		**Discretisation schemes**									
		2 vehicle categories				3 vehicle categories					
		a	b	c	d	a	b	c	d	e	f
1	(3m) (λ=0.4)	3				3	3	3			
2	(5m) (λ=0.2)		5			5			5	5	
3	(7m) (λ=0.15)			7			7		7		7
4	(9m) (λ=0.1)				9			9		9	9
5	(11m) (λ=0.05)	11	11	11	11	11	11	11	11	11	11
Expected revenue		**73.97**	58.97	36.34	32.54	**78.13**	76.14	74.37	62.20	60.17	38.30

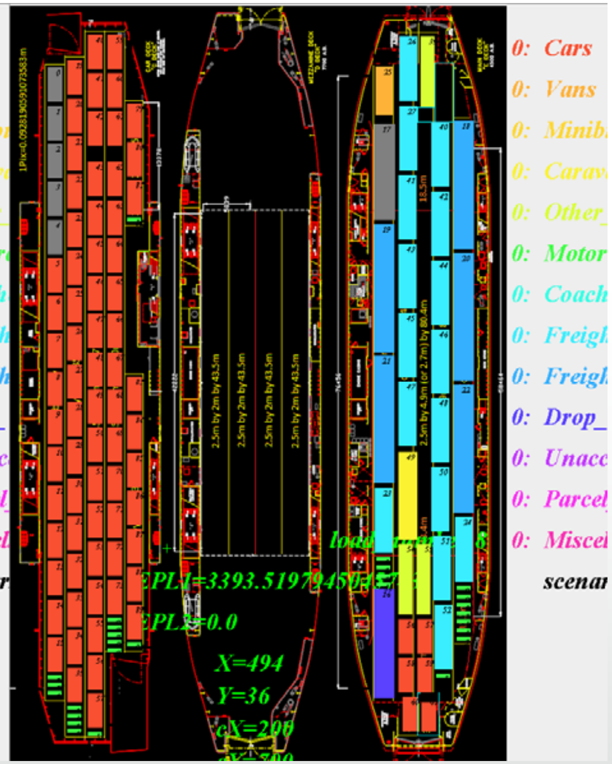
- Modelling small vehicle types with high arrival rates in detail is essential for maximising revenue
- Use as many groups of vehicles as possible (but tractability becomes a problem)
- 2 vehicle types 93%, 3 vehicle types 98%, 4 vehicle types 99%: of the optimal revenue without discretisation

Simulation based approach test instance scenarios

High car demand
2 Mezzanine decks



Medium demand
1 Mezzanine deck



High freight demand
0 Mezzanine decks

Fixed vs flexible deck configuration average revenues

Mezzanine decks	0 (fixed)	1 (fixed)	2 (fixed)	Flexible
High car demand	68.80	70.95	**71.12**	***71.61***
Medium demand	68.36	***69.24***	66.10	67.73
High freight demand	**66.56**	60.77	46.73	***66.58***
Ave DP **solution time** (seconds)	16.05	189.87	129.66	

- Identifies the best fixed deck configurations for each demand scenario
- Flexible pricing strategy is not always best
- The dynamic program can be solved very rapidly
- Choosing the best deck configuration improves revenues by an average of 17%

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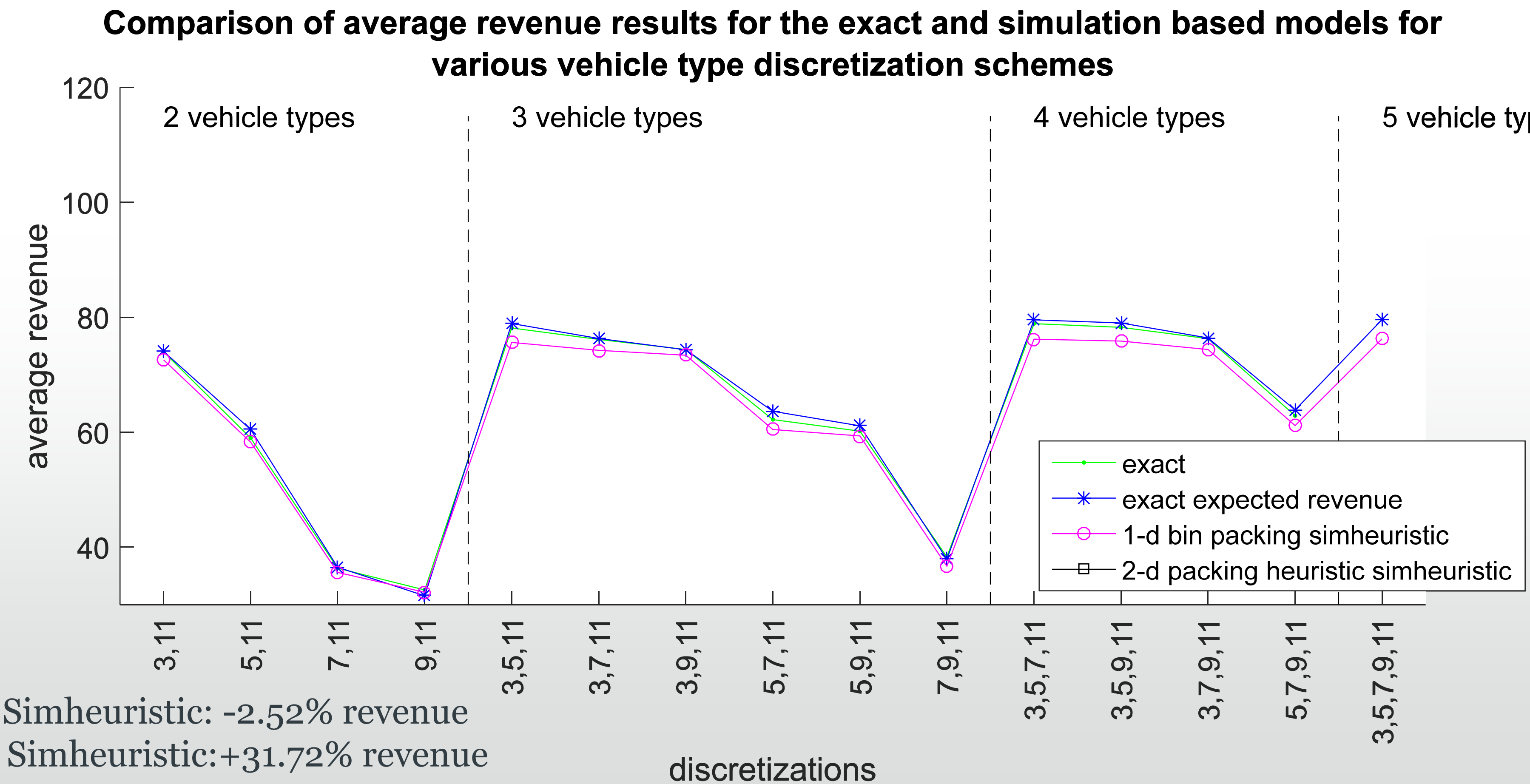
Benefit of 2-d vehicle packing (no lanes)



- 1-D Simheuristic: -2.52% revenue
- 2-D Simheuristic:+31.72% revenue

Future work

- Integrate dynamic pricing with CLV considerations
- Investigate the long term impact of optimal dynamic pricing policies
- Commercial partners are interested in their price acceptance model. The challenge is that their data is capacity constrained and click data is unreliable
- Investigate the impact of bottle necks in the loading procedure on packing feasibility

Conclusion

- The general framework for integrating packing and pricing was introduced and the main challenges involved were highlighted
- Two alternative approach—exact and simulation based—were presented
- Insights:
  - Revenue-to-go is not necessarily monotonically increasing in total remaining lane length
  - Careful vehicle type discretisation significantly improves revenues, small high demand vehicle types should be modelled in as much detail as possible
- Results
  - Finding the best deck configuration increases revenue by 17% on average
  - Simulation based approach achieve 97.48% of optimal revenue
  - Considering 2D packing increases revenues by an average of 31.72%

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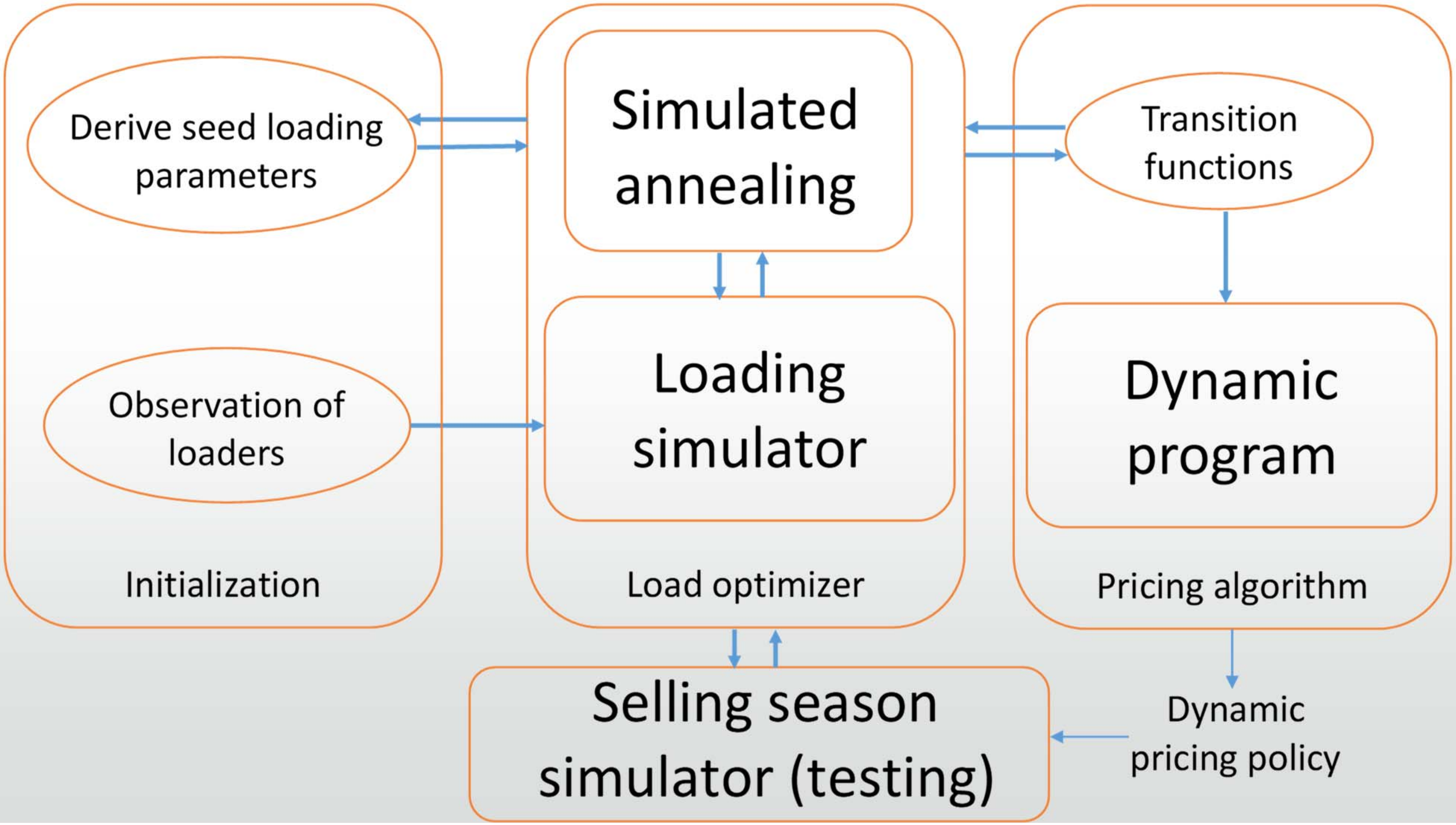
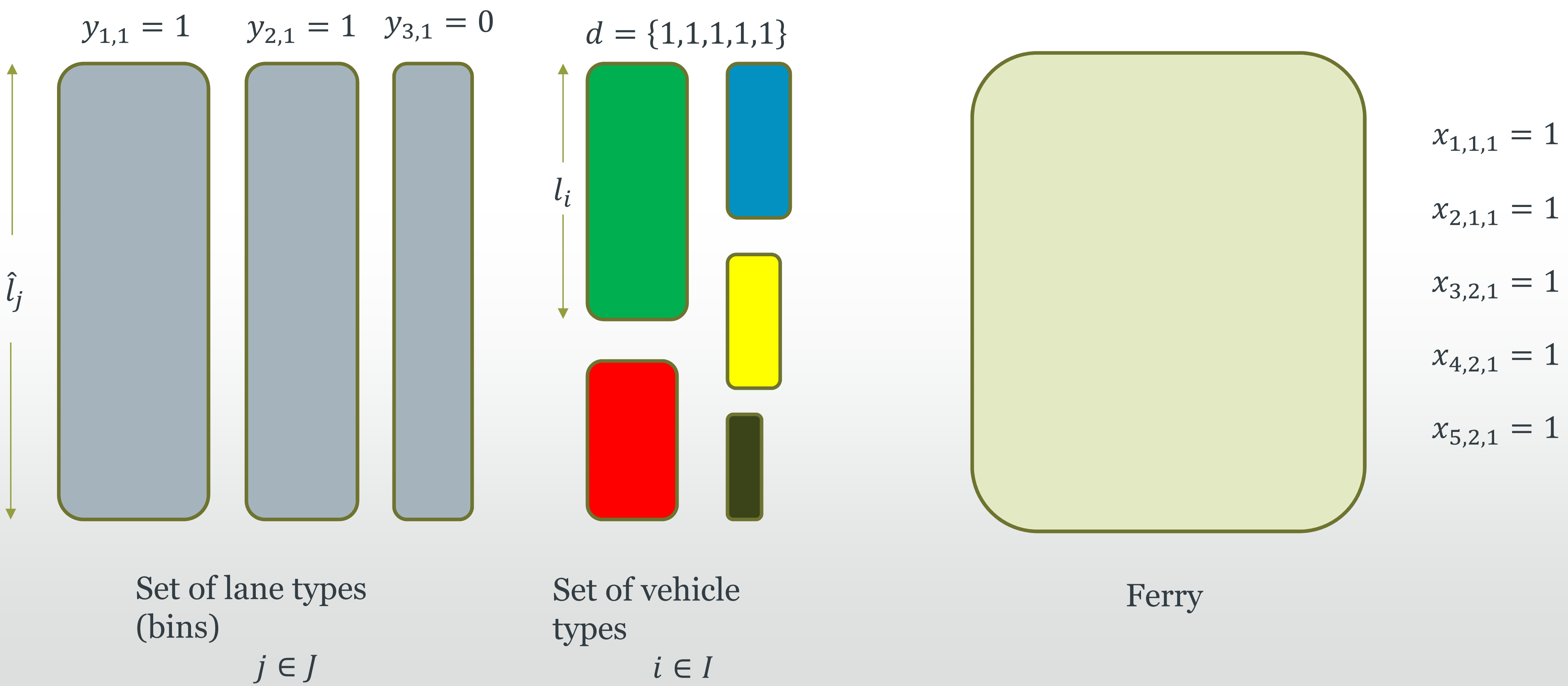
Simulation approach overview



Integer programming 1-D bin packing formulation (Exact)



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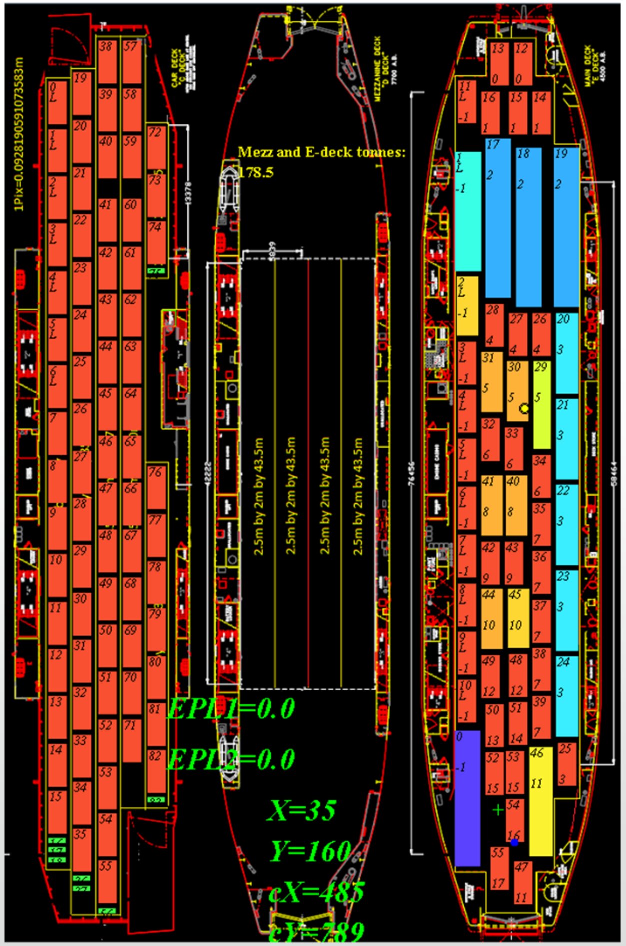
2D Packing on the main deck



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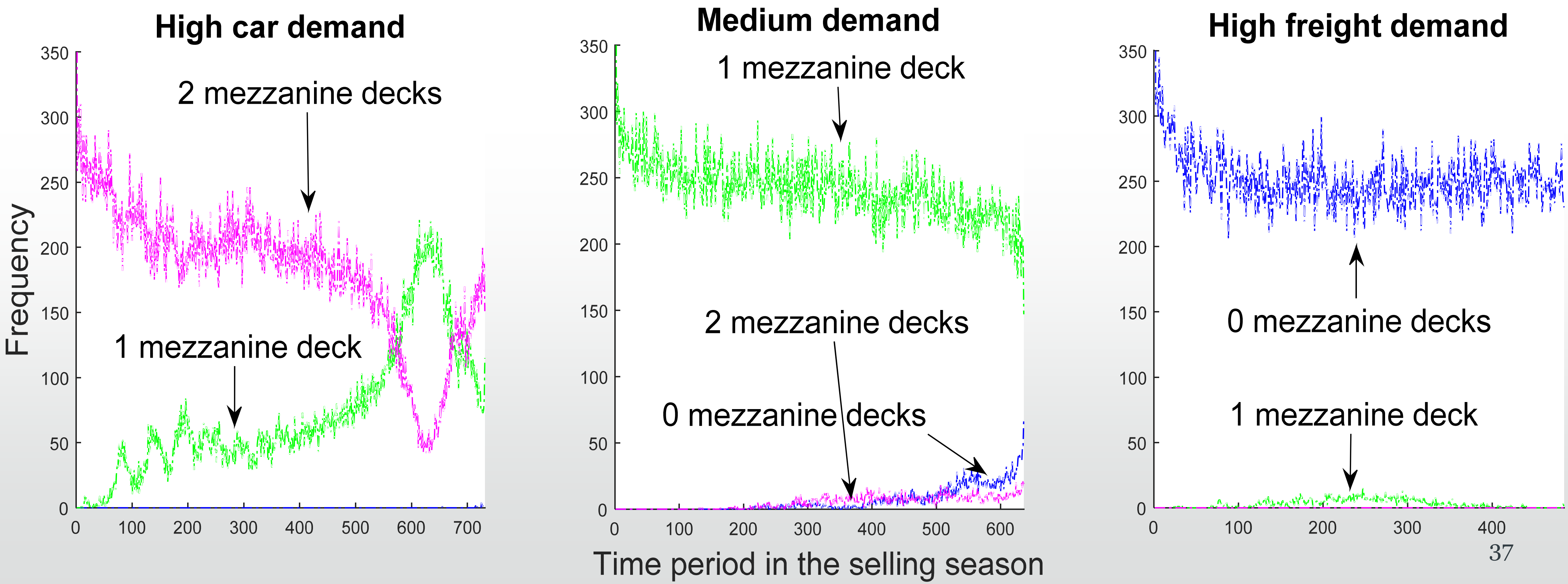
Simulation Heuristic Experiment Parameters

	Dimensions (metres)			Demand scenario arrival rates			rate of lift	parking gags	
Vehicle type	Length	Width	Height	1. High car	2. Medium	3. High freight	requirement	longitudinal	side
Car	4.326	1.871	1.5	0.88264	0.82058	0.67272	0.1	0.557	0.314
Van	6.132	2.182	2.3	0.02655	0.02468	0.02023	0	0.505	0.159
Minibus	6	2.185	2.5	0.02106	0.01958	0.01605	0.01	0.505	0.158
Caravan	11.025	2.35	2.5	0.01958	0.01821	0.01493	0.02	0.765	0.075
Other towed	8.86	1.8	2.9	0.0003	0.00701	0.02299	0	0.54	0.35
Motorcycles	0.5	1.8	1.1	0.04421	0.0411	0.0337	0	0.557	0.35
Coaches	12.064	2.633	3	0.00017	0.00395	0.01294	0.5	0.85	0
Freight medium	8.109	2.252	3.2	0.00155	0.03576	0.11728	0	0.56	0.124
Freight large	16.093	2.57	4.6	0.00082	0.01876	0.06152	0	0.463	0
Drop trailer	13.75	2.57	4	0.00034	0.00778	0.02553	0	0.463	0
Unaccompanied car	4.326	1.871	1.5	0.00061	0.00057	0.00047	0	0.557	0.314
Parcel cage	3	1.5	1.5	0.00083	0.00077	0.00063	0	0.557	0.5
Miscellaneous	7.957	2.024	2.55	0.00134	0.00124	0.00102	0	0.573	0.238

Flexible configuration pricing policy decision frequencies



The effect of packing consideration on pricing

- Value of the total remaining lane length is not monotonic (Graph 1)
- Careful discretisation of vehicle types is important (Table 1)
- In case study example optimal deck configurations are identified for different demand scenarios
- Dynamic deck configuration policies have their merits
- Simulation approach attains close to optimal revenue for in the 1-d bin packing model whilst remaining tractable for larger and more complex problem instances

Comparison of methods

Exact	Criteria	Simulation
Dynamic programming	**Pricing model**	Approximate dynamic programming
1-d bin packing (lane parking)	**Packing model**	2-d packing heuristic
Number of vehicles of each type	**State definition**	The remaining area in each distinct deck region (2 or 3 dimensions for a real world case study)
Yes	**Optimality guaranteed**	No (but close to)
1 day	**Solution time**	10 minutes
5 vehicle types	**Max problem size**	13 vehicle types handled easily
Lane parking with parking gaps included in allocated space also captures height restrictions	**Real world constraints**	Lift requirements, parking gaps, lowerable mezzanine deck height restriction, position reachability, drop trailer positions, large vehicle manoeuvrability
Packing modelled exactly in dynamic pricing and selling season	**General**	Approximates packing in dynamic pricing but exactly in the selling season